

H. Yaguchi, I. Masuda, S. Shioiri and Y. Miyake\*, YAYOICHO:

## Analysis of the Color Discrimination Data in the Physiological Based Color Space

*Discrimination thresholds in various color directions from various background colors were measured using a computer controlled color monitor with 12 bits of color resolution per primary. The discrimination thresholds were determined for five background colors, white, red, yellow, green and blue. Thresholds for 27 directions in a color space were determined for the test stimulus from each background color by the interleaved staircase method. The experimental data were analyzed in the cone excitation space. The discrimination thresholds obtained by the experiment were plotted in the orthogonal space whose axes correspond to the Weber's fraction of the cone excitation, which are  $\Delta L/L$ ,  $\Delta M/M$  and  $\Delta S/S$  for the long-, middle- and short-wavelength sensitive cones, respectively. As a result, discrimination thresholds for all five background colors in a given spatial condition were roughly located on the surface of an identical ellipsoid in the ( $\Delta L/L$ ,  $\Delta M/M$ ,  $\Delta S/S$ ) space. The evidence obtained by the present study strongly supports the line element theory of color discrimination.*

*A l'aide d'un moniteur couleur, d'une résolution de 12 bits par couleur primaire, les seuils de discrimination furent mesurés selon plusieurs directions, tout en étant superposés à diverses couleurs de fond. Les seuils de discrimination furent déterminés pour cinq couleurs de fond, à savoir le blanc, le rouge, le jaune, le vert et le bleu. Les seuils furent déterminés dans l'espace chromatique pour un ensemble de 27 directions pour le test stimulus et ceci pour chaque couleur de fond en procédant selon la méthode des escaliers intercalés (Interleaved Staircase). Les résultats expérimentaux furent analysés dans l'espace d'excitation des cônes. Les seuils de discrimination ainsi obtenus furent tracés dans un espace orthogonal, dont les axes associés aux fractions de Weber de chaque cône  $\Delta L/L$ ,  $\Delta M/M$  et  $\Delta S/S$  correspondent aux cônes dont la sensibilité est associée respectivement aux longueurs d'onde longues, moyennes et courtes. Comme résultats, les seuils de discrimination pour les cinq couleurs de fond, selon une condition spatiale donnée, furent approximativement localisés sur la surface d'un ellipsoïde de l'espace  $\Delta L/L$ ,  $\Delta M/M$  et  $\Delta S/S$ . Les résultats obtenus, lors de la présente étude, tendent à supporter fortement la théorie de l'élément de longueur de la discrimination des couleurs.*

*Mittels eines rechnergesteuerten Farbmonitors mit einer Auflösung von 12 bit pro Primärfarbe wurden bei den fünf Umfeldfarben Weiß, Rot, Gelb, Grün und Blau Farbunterscheidungsschwellen in verschiedenen Richtungen des Farbenraumes gemessen. Für jede Umfeldfarbe wurden Schwellen in 27 Richtungen des Farbenraumes mit Hilfe der Treppenhausmethode bestimmt. Die experimentellen*

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\* Chiba University

*Daten wurden im Raum der Zapfenanregung analysiert. Die Darstellung der Unterscheidungsschwellen erfolgt im orthogonalen Raum, dessen Achsen dem Weberschen Anteil der Zapfenanregung entsprechen, d. h.  $\Delta L/L$ ,  $\Delta M/M$  und  $\Delta S/S$  für die lang-, mittel- und kurzwellig empfindlichen Zapfen. Es zeigte sich, daß für gegebene räumliche Bedingungen die Unterscheidungsschwellen im  $(\Delta L/L, \Delta M/M, \Delta S/S)$ -Raum für alle fünf Umfeldfarben ungefähr auf demselben Ellipsoid liegen. Die Ergebnisse unterstützen die Gültigkeit der Theorie der Linienelemente für die Farbunterscheidung.*

## 1. Introduction

It is very important to take account of how small color difference we can discriminate in setting and interpreting production tolerances. The threshold detection of small color differences has been the subject of greatest study during past years. The most well-known quantitative analysis of color discrimination is MACADAM's work [1]. The MACADAM's ellipses were consulted to establish the CIE color difference formula. However, the MACADAM's ellipses are standard deviations derived from repeated complete color matches and not just noticeable color differences. The more straightforward method to estimate color discrimination is measuring a detection threshold of color difference. One of the example is STILES' classical study on the twocolor increment threshold [2], where a test flash is detected on a uniform adapting field.

Thanks to advance of the raster display system, it becomes easy to control the presentations of stimuli such as a spatial and/or temporal profile of test stimulus and the increment direction in a color space. One of purposes of the present study is to investigate how detection of small color difference is affected by the spatial profile of the test stimulus presentation.

Another interest is to study the physiological mechanism of the color discrimination. The theoretical approach to predicting the discriminability of the two colors was first introduced by HELMHOLTZ, known as *line element theory* [3]. We analyzed the color discrimination data in the physiological based space in order to see how line element theory predicts the experimental data.

## 2. Experiments

The test patterns which the subject views are generated on the screen of a color video monitor. The color of pattern components is controlled by a color map which has 12 bits of resolution of each primary; that is, there are 4096 discrete colors between the minimum and the maximum output of each primary color.

The experiment was done by using the method developed by COWAN et al. [4]. In this experiment, the observer sees a brief flash of the test

stimulus with a color slightly different from a steady background color. The background field was a square whose size was 6 degrees by 6 degrees. The test stimulus was presented in any one of four panes, each of them 1 degree by 1 degree, aligned as a 2 by 2 matrix with a fixation point in its center. The observer's task was to report which of the four panes contained the test stimulus. The test patterns were presented in three different spatial conditions; (a) the edge condition in which a border between the test stimulus and the background consisted of an edge, (b) the gap condition in which the test stimulus and the background were separated with a black gap of 6 minute width, and (c) the noise condition in which a random dot pattern, each dot 6 minutes by 6 minutes, was overlaid upon the edge condition. The stimuli are shown schematically in Fig. 1.

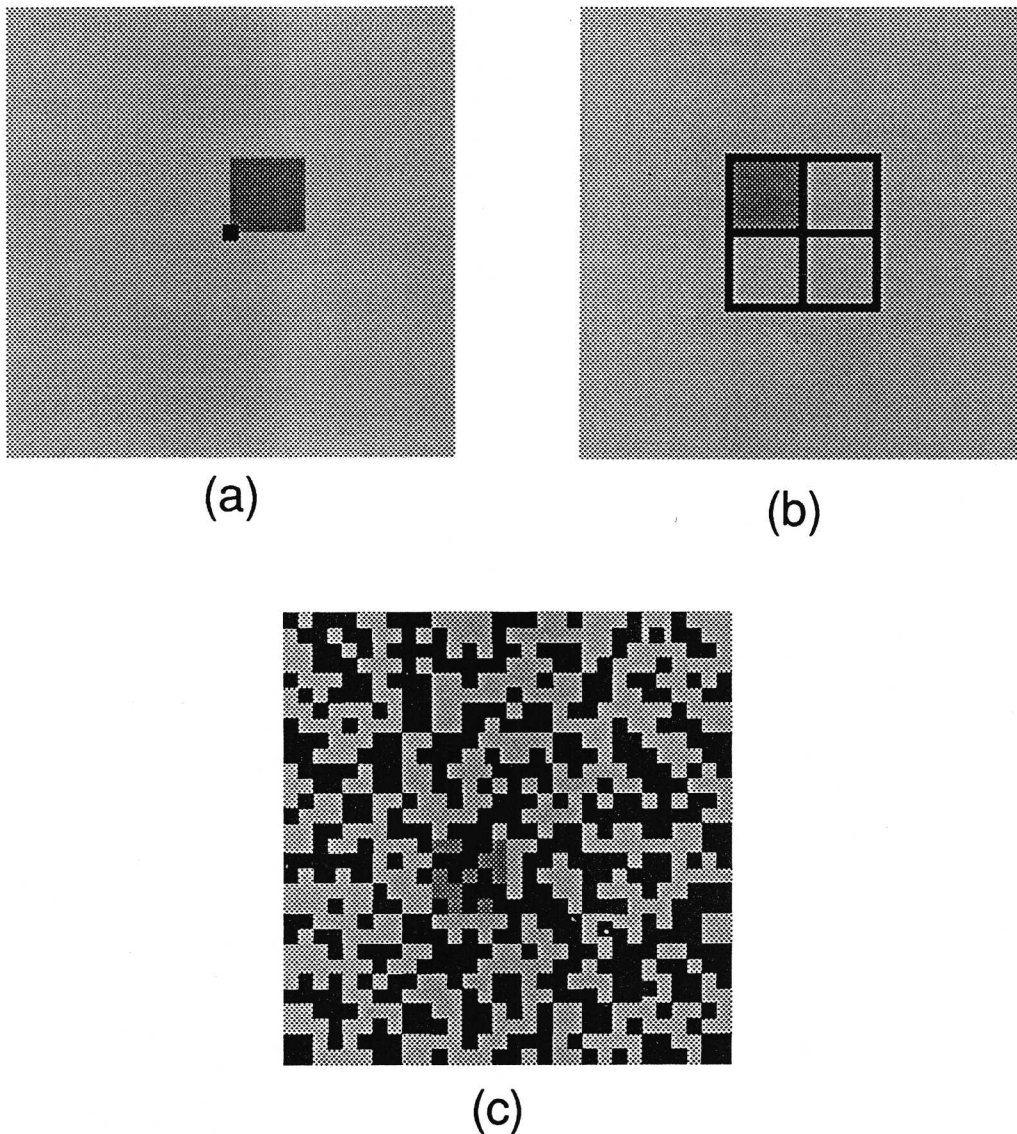


Fig. 1: The schematic views of the test pattern.

**Table 1:** The chromaticity coordinates of background colors

Background colors	$x$	$y$
white	0.313	0.329
red	0.401	0.317
green	0.298	0.469
blue	0.189	0.140
yellow	0.364	0.449

The temporal profile of the test stimulus was a 200 msec duration rectangle for all conditions. The background color was generated by a fixed mixture of the video monitor's red, green and blue primaries. The increment and decrement stimuli were generated by adding or subtracting from the background color appropriate amounts of the red, green and blue. The discrimination thresholds were determined for five background colors, white, red, yellow, green and blue. The CIE  $xy$  chromaticity coordinates

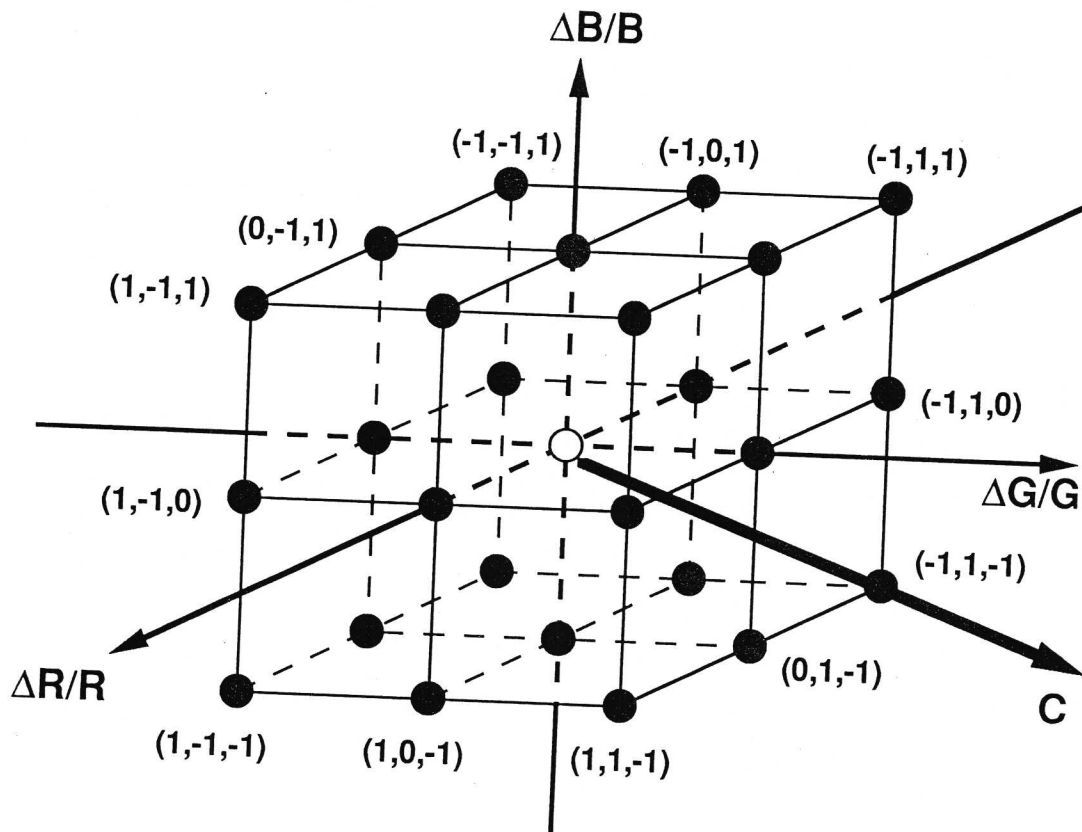


Fig. 2: Schematic diagram showing the 26 directions in the luminance contrast space.

dinates are shown in Table 1. The luminance of background color was set to 15 cd/m<sup>2</sup> for all.

The test stimuli are expressed in terms of the luminance contrast, that is, the increment or decrement luminance of test stimulus ( $\Delta R, \Delta G, \Delta B$ ) is divided by the luminance of the primary color component of the background color ( $R, G, B$ ). Thresholds for the test stimuli along 26 different directions away from each background color were measured. Fig. 2 represents the schematic diagram showing the 26 directions of the test stimulus in the luminance contrast space. When measuring the threshold for a given direction in a color space, the red/green/blue ratio of the test stimuli was kept constant. Threshold values are given by the EUCLIDEAN distance, that is,  $[(\Delta R/R)^2 + (\Delta G/G)^2 + (\Delta B/B)^2]^{1/2}$ . Thresholds were determined by the interleaved staircase method. In a single session, thresholds were measured using from the same background color, but

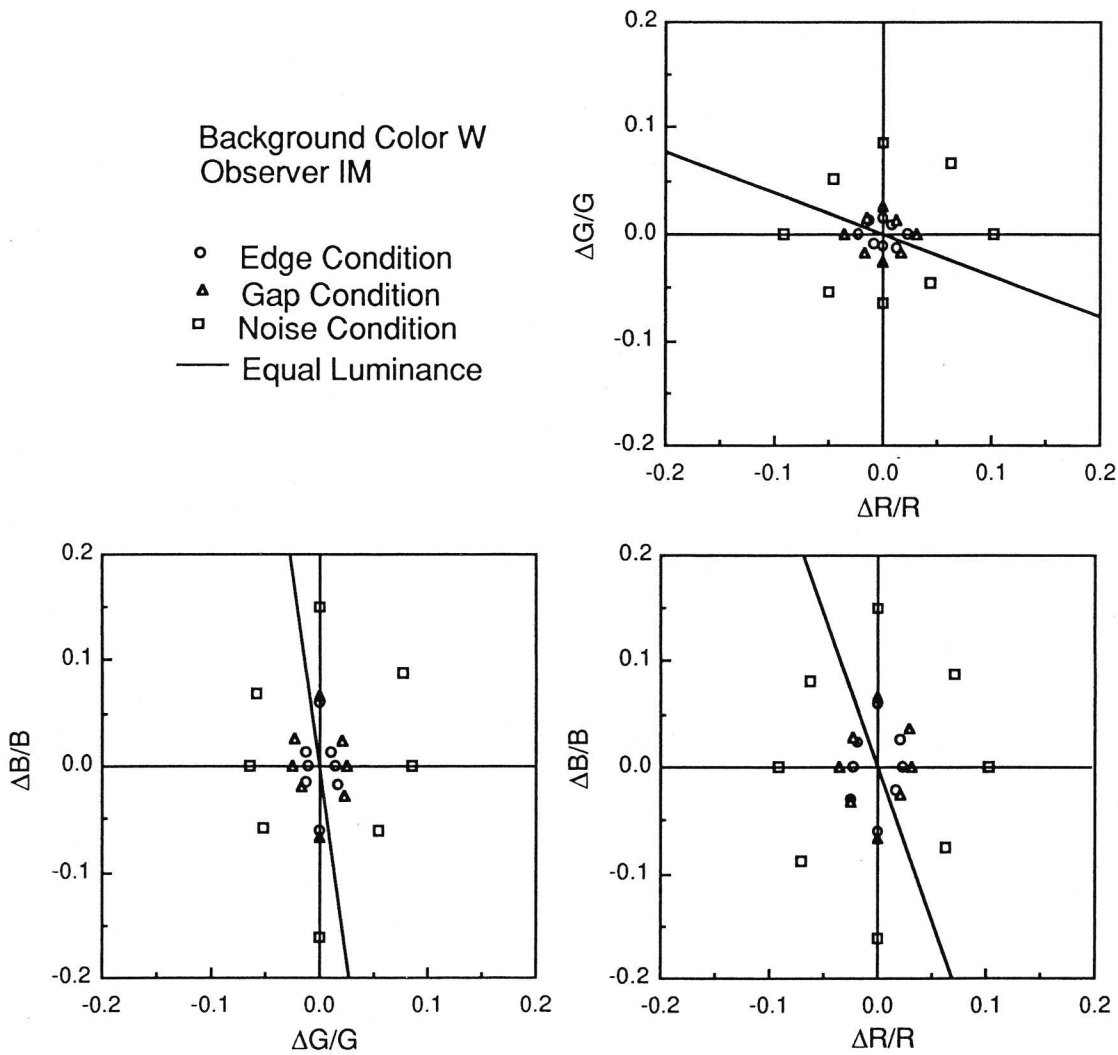


Fig. 3: Detection thresholds for the white background in the luminance contrast space.



going in different color space directions. Thresholds for four directions were measured in a session.

### 3. Results

The thresholds obtained in the experiment are plotted in the luminance contrast space. Fig. 3 shows the results for observer IM for the directions lying within the three planes of the color space;  $\Delta R/R - \Delta G/G$ ,  $\Delta R/R - \Delta B/B$ , and  $\Delta G/G - \Delta B/B$  plane. The center of each graph corresponds to the background color, in this case, white. Different symbols indicate the different observing conditions. The straight line shows the isoluminance line. As expected, the thresholds for the noise condition (open squares) are much larger than the other conditions. Fig. 4 shows the thresholds for the green background. If we compare the thresholds for the gap condition (open triangles) and those for the edge conditions

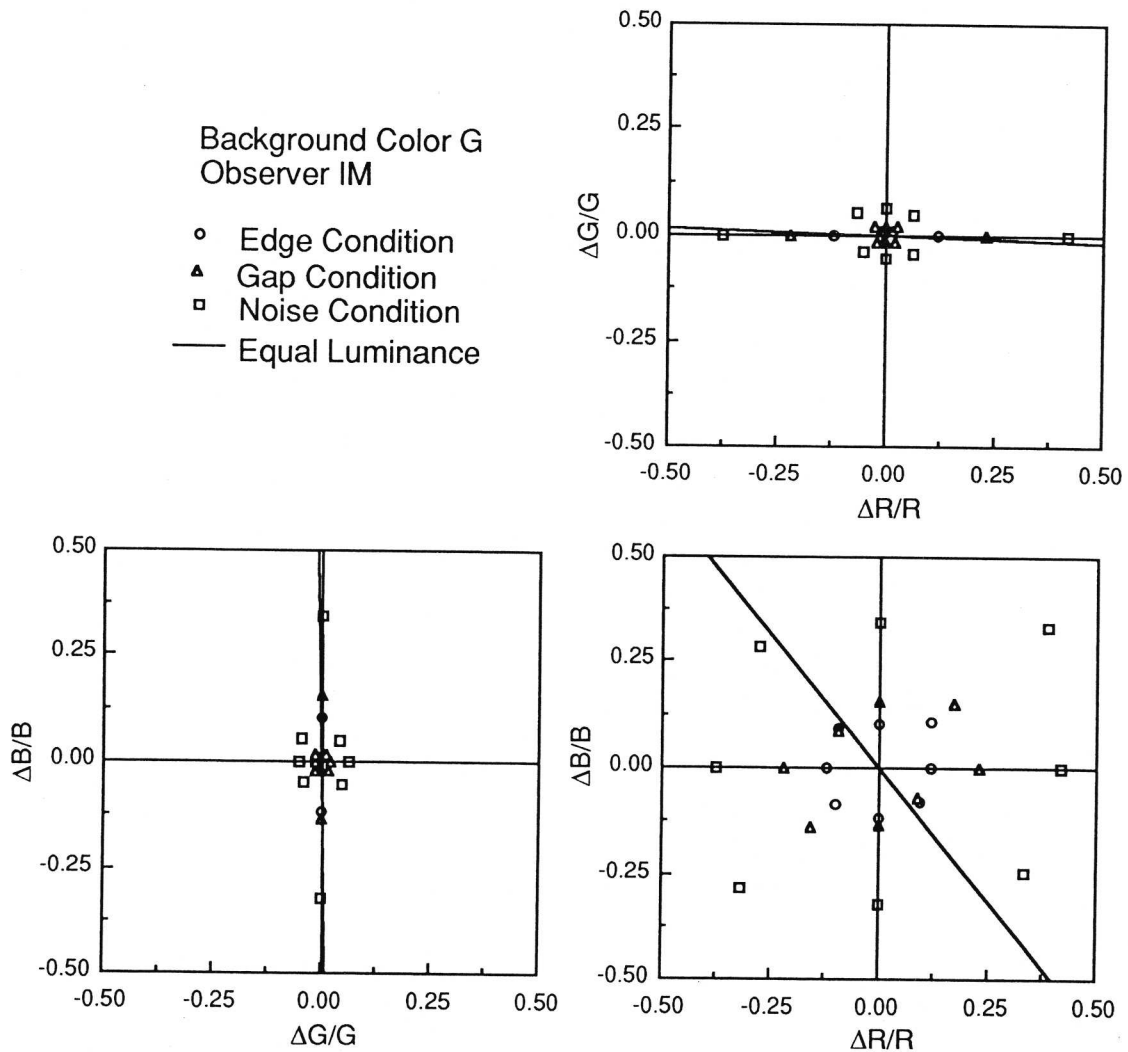


Fig. 4: Same as Fig. 3 but for the green background.

(open circles), the graph of the  $\Delta R/R - \Delta B/B$  plane shows that the gap condition increased the threshold toward the direction of perpendicular to the isoluminance line but did not affect much along the isoluminance direction.

#### 4. Line element theory

We analyzed the present experimental results according to HELMHOLTZ's line-element theory. HELMHOLTZ assumed that the smallest discriminable difference is obtained by combining the fractional deviations for the excitation of the three cone classes, the long-wavelength-sensitive (L), middle-wavelength-sensitive (M), and short-wavelength-sensitive (S) cone. This gives the line element given by a square root of a sum of squares of the cone contrast, that is

$$\partial s = [(\Delta L/L)^2 + (\Delta M/M)^2 + (\Delta S/S)^2]^{1/2}, \quad (1)$$

where  $\Delta L$ ,  $\Delta M$  and  $\Delta S$  are the change in cone excitation in the L, M, and S cones due to the increment or decrement test stimulus, and  $L$ ,  $M$ , and  $S$  are the cone excitation due to the background color. However, the line element may be more complicated functions of the cone contrast. More general form of the line element may be given by the distance in a three-dimensional RIEMANNIAN as following quadratic equation,

$$(\partial s)^2 = Q_{11}(\Delta L/L)^2 + 2Q_{12}(\Delta L/L) (\Delta M/M) + Q_{22}(\Delta M/M)^2 + 2Q_{23}(\Delta M/M) (\Delta S/S) + Q_{33}(\Delta S/S)^2 + 2Q_{31}(\Delta S/S) (\Delta L/L). \quad (2)$$

The equation with a constant  $\partial s$  makes the locus an ellipsoid. The coefficients  $Q_{ij}$  determine the directions and lengths of semiaxis of the ellipsoid.

The excitations of L-cone, M-cone, and S-cone for each test stimulus are calculated by the following equations,

$$L = \int E(\lambda) l(\lambda) d\lambda \quad (3)$$

$$M = \int E(\lambda) m(\lambda) d\lambda \quad (4)$$

$$S = \int E(\lambda) s(\lambda) d\lambda. \quad (5)$$

where  $E(\lambda)$  is the spectral radiance of the stimulus, and  $l(\lambda)$ ,  $m(\lambda)$  and  $s(\lambda)$  are the spectral sensitivity functions for L-cone, M-cone, and S-cone, respectively, derived by SMITH and POKORNY [5], expressed as the linear combination of the color matching functions modified by JUDD, as follows,

$$\begin{bmatrix} l(\lambda) \\ m(\lambda) \\ s(\lambda) \end{bmatrix} = \begin{bmatrix} 0.15514 & 0.54312 & -0.03286 \\ -0.15514 & 0.45684 & 0.03286 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} \quad (6)$$

The threshold data obtained by the present study were plotted in the cone contrast space, that is the  $(\Delta L/L, \Delta M/M, \Delta S/S)$  space. Fig. 5 shows the detection thresholds in the cone contrast space. The data used in this graph are derived from those shown in Fig. 3, but plotted points show the projection to each plane. It is clearly shown that the discriminability along the direction of the excitation contrast of S-cone is poorer than those of M-cone and L-cone. Fig. 6 shows thresholds for all background colors plotted altogether in a common  $\Delta L/L - \Delta M/M$  plane. Different symbols indicate different background color. Figs. 6(a), 6(b), 6(c) correspond to three different observing conditions; the edge condition, the gap condition, and the noise condition, respectively. These results show that the locus of the thresholds were roughly located in an identical ellipse on the  $\Delta L/L - \Delta M/M$  plane for each observing condition. We calculated the

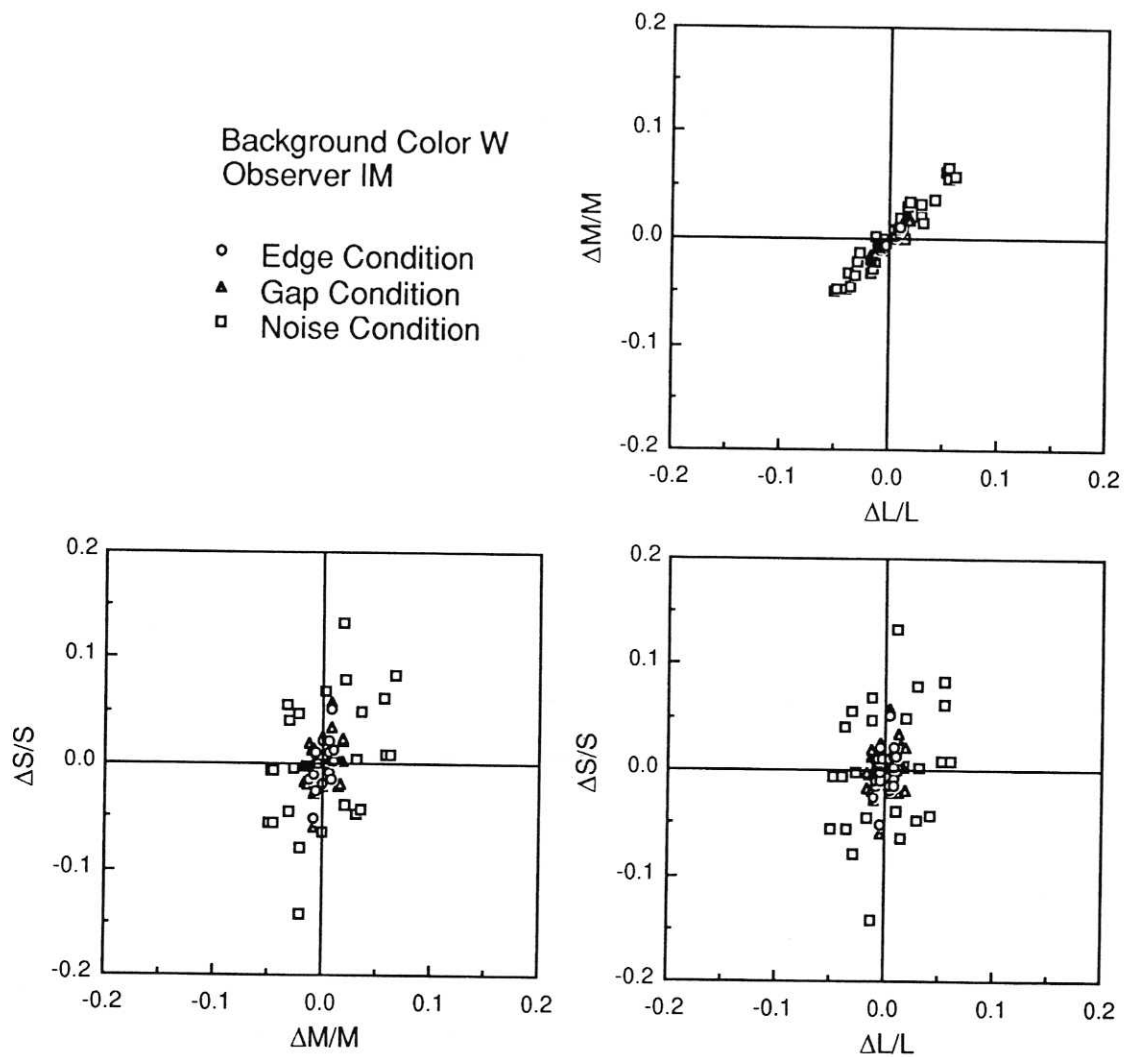


Fig. 5: Thresholds in the cone contrast space.



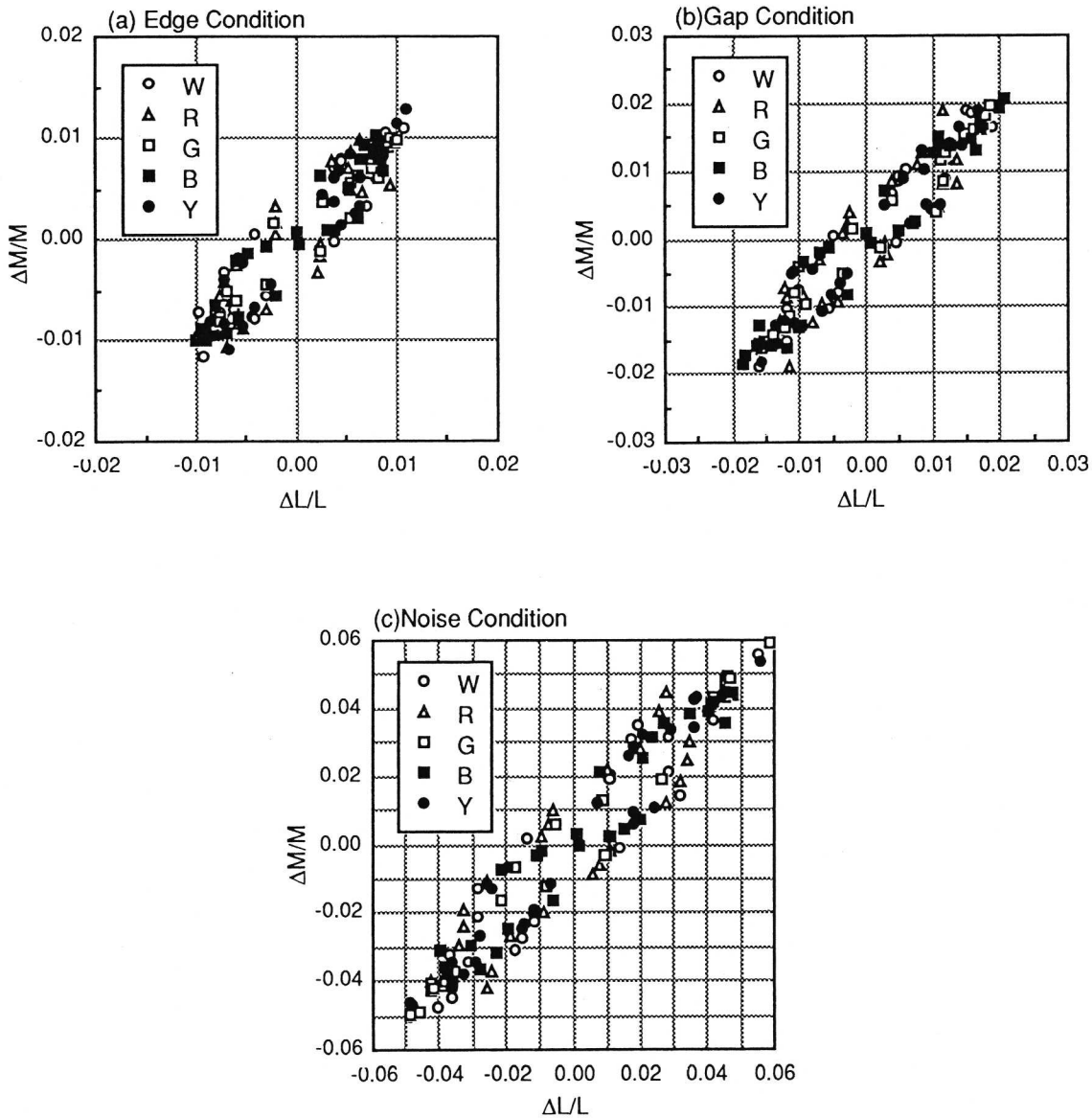
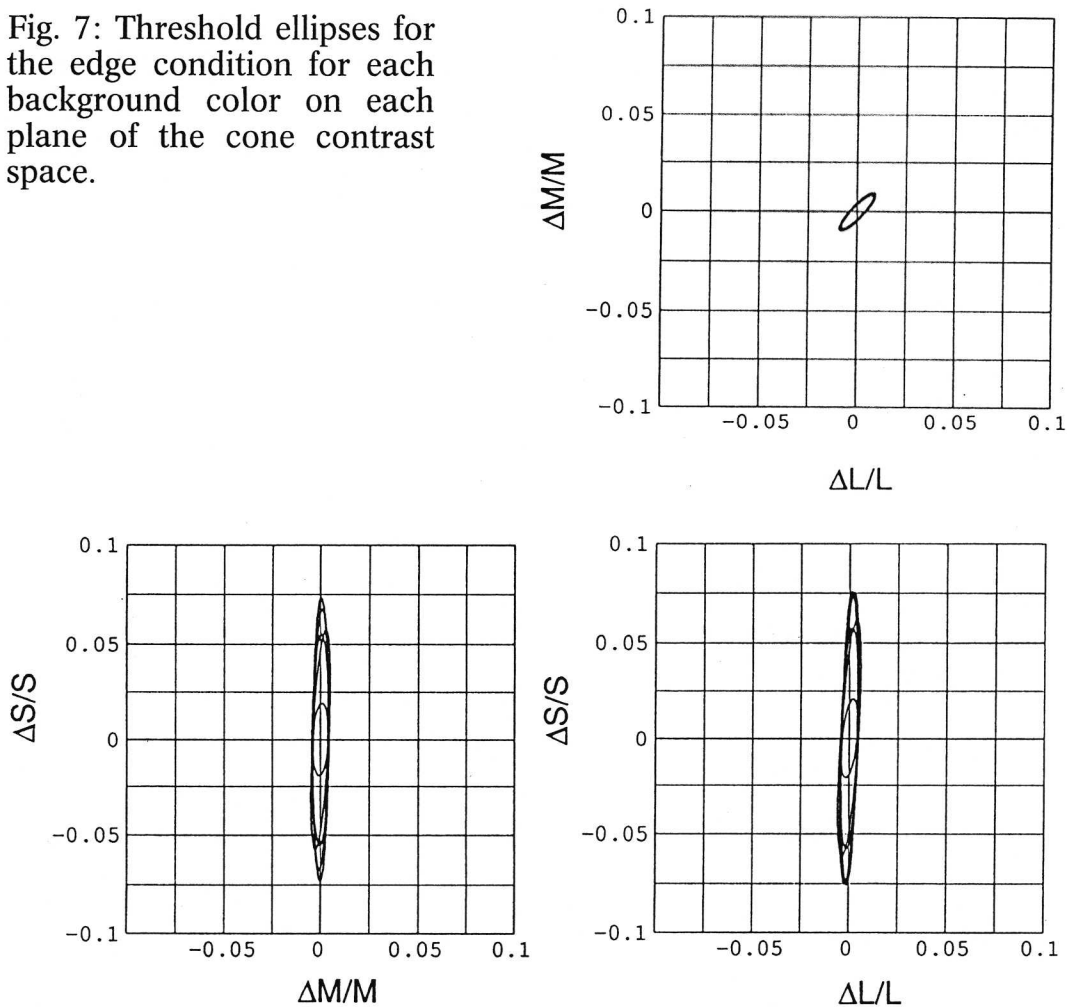


Fig. 6: Thresholds for all background colors on the  $\Delta L/L - \Delta M/M$  plane.

coefficients  $Q_{ij}$  of Eq. (2) from the experimental data points for each background color by a multiple regression analysis. The calculated ellipsoid for the edge condition for each background color is presented as its cross section with each plane in Fig. 7. The ellipses on the  $\Delta L/L - \Delta M/M$  plane coincide with each other. On the other hand, thresholds along the direction of  $\Delta S/S$  are deviated among the different background colors. From these results, it is suggested that the color discrimination by the L-cone and M-cone obeys the WEBER law, but the discrimination by the S-cone does not.

Fig. 7: Threshold ellipses for the edge condition for each background color on each plane of the cone contrast space.



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*Authors' address:*

*Dr. Hirohisa Yaguchi*

*Dr. Isao Masuda*

*Dr. Satoshi Shioiri*

*Dr. Yoichi Miyake*

*Department of Image Science and Engineering*

*Chiba University*

*1-33 Yayoicho, Inage-Ku*

*Chiba 263*

*Japan*

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